Sheet .1.

1 Find $e(0)$ and $e(10)$ for

$$E(z) = \frac{1}{(z-1)(z-0.3)}$$ using Partial Fraction

then check $e(0)$ using initial value

Sol^n

using Partial Fraction

$$E(z) = \frac{A_1}{z-1} + \frac{A_2}{z-0.3} = z^{-1}\left[\frac{A_1 z}{z-1} + \frac{A_2 z}{z-0.3}\right]$$

By inspection

$$A_1 = 1/0.7 = 1.43 \quad , \quad A_2 = 1/-0.7 = -1.43$$

$$\therefore \quad E(z) = z^{-1}\left[\frac{1.43 z}{z-1} - \frac{1.43 z}{z-0.3}\right]$$

$z^{-1}T \longrightarrow$

$$e(k) = 1.43\,u(k-1) - 1.43\,(0.3)^{k-1}\,u(k-1) \quad \#$$

$$\therefore\ e(0) = 1.43\,u(-1) - 1.43\,(0.3)^{-1}\,u(-1) = \text{Zero}$$

$$e(10) = 1.43\,u(9) - 1.43\,(0.3)^{9}\,u(9) = 1.4286$$

Check of $e(0)$ using initial value

$$e(0) = \lim_{t\to 0} e(t) = \lim_{z\to\infty} E(z)$$

$$= \lim_{z\to\infty} \frac{1}{(z-1)(z-0.3)} = \frac{1}{\infty} = \text{Zero}$$

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2 Given: $e(t) = A\cos(\omega t)$, $E(z) = \dfrac{3z(z-0.6967)}{z^2 - 1.3934z + 1}$

Required: (with $T = 0.2\,s$) $\rightarrow$ find A, ω

Sol^n

$$e(kT) = A\cos(\omega kT) \longrightarrow E(z) = \frac{Az(z-\cos(\omega T))}{z^2 - 2z\cos(\omega T) + 1}$$

from the given $E(z)$

$$\therefore \boxed{A = 3}$$

$$\therefore \cos(\omega T) = 0.6967 \longrightarrow \omega = \frac{\cos^{-1}(0.6967)}{T}$$ 0.8

using radians $\quad \therefore \quad \omega = \dfrac{0.8}{0.2} = \boxed{4 \text{ rad/s}}$

6 Given $y(k) - y(k-1) + y(k-2) = u(k)$

where $u(k) = 1$ for $k \geq 0$, $T = 1s$

Required: i) Find $y(0), y(1), y(2)$ and $y(3)$ using Power method

ii) Verify Part i by solving difference equation directly

iii) Solve $y(k)$ using Z-transform

Sol^n

$$y(k) - y(k-1) + y(k-2) = u(k)$$

taking Z-transform

$$Y(Z) - Z^{-1}Y(Z) + Z^{-2}Y(Z) = U(Z)$$

$$Y(Z)(1 - Z^{-1} + Z^{-2}) = \frac{Z}{Z-1}$$

$$Y(Z) = \frac{Z}{(Z-1)(1-Z^{-1}+Z^{-2})} = \frac{Z^3}{(Z-1)(Z^2-Z+1)}$$

i) Using power method (Long division)

$$1 + 2Z^{-1} + 2Z^{-2} + Z^{-3} + Z^{-6}$$

$Z^3 - 2Z^2 + 2Z - 1 \;\big)\; Z^3$

$$\cancel{Z^3} - 2Z^2 + 2Z - 1$$

$$\cancel{2Z^2} - 2Z + 1$$

$$\cancel{2Z^2} - 4Z + 4 - 2Z^{-1}$$

$$\cancel{2Z} - 3 + 2Z^{-1}$$

$$\cancel{2Z} - 4 + 4Z^{-1} - 2Z^{-2}$$

$$\cancel{1} - 2Z^{-1} + 2Z^{-2}$$

$$1 - 2Z^{-1} + 2Z^{-2} - Z^{-3}$$

$$Z^{-3}$$

$$Z^{-3} - 2Z^{-4} + 2Z^{-5} - Z^{-6}$$

$y(0) = 1$
$y(1) = 2$
$y(2) = 2$
$y(3) = 1$
$y(4) = 0$
$y(5) = 0$
$y(6) = 1$

Note:
for $Y(Z) = 1 + 2Z^{-1} + 2Z^{-2} + Z^{-3} + \cdots$

knowing that
$$Y(Z) = \sum_{k=0}^{\infty} y(k) Z^{-k}$$

$\therefore Y(Z) = y(0) + y(1)Z^{-1} + y(2)Z^{-2} + \cdots$

ii) $\because \quad y(k) - y(k-1) + y(k-2) = u(k)$

$\therefore \quad y(k) = y(k-1) - y(k-2) + u(k)$

For $k=0 \longrightarrow y(0) = \cancel{y(-1)} - \cancel{y(-2)} + u(0) = 1$

For $k=1 \longrightarrow y(1) = y(0) - \cancel{y(-1)} + u(1) = 2$

For $k=2 \longrightarrow y(2) = y(1) - y(0) + u(2) = 2$

For $k=3 \longrightarrow y(3) = y(2) - y(1) + u(3) = 1$

For $k=4 \longrightarrow y(4) = y(3) - y(2) + u(4) = 0$

$\vdots$

iii) Using Z-transform

$$Y(Z) = \frac{Z^3}{(Z-1)(Z^2-Z+1)} = Z\left[\frac{Z^2}{(Z-1)(Z^2-Z+1)}\right]$$

$$= Z\cdot\left[\frac{A}{Z-1} + \frac{BZ+C}{Z^2-Z+1}\right] = \frac{Z}{Z-1} + \frac{Z}{Z^2-Z+1}$$

we know that

$$Z\left[\sin(ak)\right]_{T=1} = \frac{Z\sin(a)}{Z^2-2Z\cos(a)+1}$$

$$\therefore \quad Y(Z) = \frac{Z}{Z-1} + \frac{Z\sin(a)}{(Z^2-Z+1)\sin(a)} \qquad \left(\text{we multiplied by } \frac{\sin a}{\sin a}\right)$$

By analogy : $2\cos a = 1 \longrightarrow \cos a = 0.5 \longrightarrow a = \pi/3$ rad

$\therefore \sin a = \sin(\frac{\pi}{3}) = \sqrt{3}/2$

$$\therefore \quad Y(Z) = \frac{Z}{Z-1} + \frac{2}{\sqrt{3}}\,\frac{Z\sin(a)}{Z^2-2\cos(a)Z+1}$$

$$\therefore \quad \boxed{y(k) = u(k) + \frac{2}{\sqrt{3}}\sin\left(\frac{\pi}{3}k\right)}$$

check

$y(0) = 1 \quad , \quad y(1) = 1 + \frac{2}{\sqrt{3}}\sin(\frac{\pi}{3}) = 2$

$y(2) = 1 + \frac{2}{\sqrt{3}}\sin(\frac{2\pi}{3}) = 2$

$y(3) = 1 + \frac{2}{\sqrt{3}}\underbrace{\sin(\pi)}_{=0} = 1$